Grade 8 Mathematics, Quarter 1, Unit 1.1

Transforming Geometric Figures Using Physical Models

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Understand congruence using physical models, transparencies, or geometry software.
- Understand similarity using physical models, transparencies, or geometry software.
- Verify the properties of rotations, reflections, and translations by sketching graphs.
- Justify (experimentally) lines taken to lines, and line segments taken to line segments of the same length.
- Validate (experimentally) congruence of angles taken to angles of the same measure.

Mathematical practices to be integrated

Model with mathematics.

- Write a geometric sequence to model rotations, reflections, translations, and dilations.
- Use manipulates to understand congruency and similarity between two-dimensional figures.

Use appropriate tools strategically.

- Use physical models, transparencies, and geometry software to describe sequences of rotations, reflections, translations, and dilations.

Essential questions

- How do you verify the properties of reflections, rotations, and translations?
- How do you compare and contrast congruence and similarity?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure.

c. Parallel lines are taken to parallel lines.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In kindergarten, students began to analyze and compare two- and three-dimensional shapes. In grades 1, 2, and 3, students reasoned with shapes and their attributes. In grade 4, students identified and classified by properties of their lines and angles. In grade 5, students classified two-dimensional figures into categories based on their properties and graph points on the coordinate plane. In grade 6, students began to draw polygons in the coordinate plane. In grade 7, students used proportional reasoning when they analyzed scale drawings.

Current Learning

Understanding congruence and similarity is a major cluster in eighth grade. Students use geometry software and physical models to verify the properties of rotations, reflections, and translations. Students sketch lines and angles by hand to demonstrate the properties of congruence and similarity.

Future Learning

The progression from congruence, to area, to similarity, will give footing to the assumption that congruent figures have the same area in algebra 1. The angle similarity will be used to develop the trigonometry of right triangles. Lines and their translations will be used to model real-world situations in economics and manufacturing.

Additional Findings

Transformations may be a challenge to some students if they have not been exposed to physically manipulating shapes. Having experiences with the physical manipulation of shapes will allow students to attain a better grasp of transformations and congruence. (*Adding It Up*, pp. 284–287)

Grade 8 Mathematics, Quarter 1, Unit 1.2

Proving Congruence and Similarity of Two-Dimensional Figures

Overview

Number of instructional days: 6 (1 day = 45–60 minutes)

Content to be learned

- Understand that a two-dimensional figure is congruent to another through a sequence of rotations, reflections, and translations.
- Describe the series of steps that shows the congruence between two-dimensional figures.
- Interpret the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- Recognize that a two-dimensional figure is similar to another through a sequence of rotations, reflections, translations, and dilations.
- Develop the series of steps that shows the similarity between two-dimensional figures.

Mathematical practices to be integrated

Model with mathematics.

- Write a geometric sequence to model rotations, reflections, translations, and dilations.
- Use manipulatives to understand congruency and similarity between two-dimensional figures.

Use appropriate tools strategically.

- Use physical models, transparencies, and geometry software to describe sequences of rotations, reflections, translations, and dilations.

Essential questions

- What are the characteristics of congruent two-dimensional figures?
- What are the characteristics of similar two-dimensional figures?
- What are the similarities and differences between congruence and similarity?
- How are dilations different from translations, rotations, and reflections on two-dimensional figures using coordinates?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**8.G**

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In kindergarten, students began to analyze and compare two- and three-dimensional shapes. In grades 1, 2, and 3, students reasoned with shapes and their attributes. In grade 4, students identified and classified by properties of their lines and angles. In grade 5, students classified two-dimensional figures into categories based on their properties and graphed points on the coordinate plane. In grade 6, students began to draw polygons in the coordinate plane. In grade 7, students used proportional reasoning when they analyzed scale drawings.

Current Learning

Understanding congruency and similarity using physical models, transparencies, or geometry software is a major cluster in grade 8. Students understand that a two-dimensional figure is congruent/similar to another by a sequence of rotations, reflections, translations, and dilations. They describe a sequence that exhibits the congruence between them. Students describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. Students must do work with congruency and similarity before they are able to justify the connections among proportional relationships, lines, and linear equations.

Future Learning

Congruency and similarity will be studied in depth in geometry. Students will begin to experiment with transformations in the plane, understand congruence in terms of rigid motions, prove geometric theorems, and make geometric constructions. Students will also continue to work with understanding similarity in terms of similarity transformations and begin to prove theorems involving similarity. All these mathematical concepts are involved in using computer animations, engineering, art, and architecture.

Additional Findings

Understanding similarity is difficult for students at this grade level. According to *Principles and Standards for School Mathematics*, "Work with magnifications and contractions, called dilations, can support students' developing understanding of similarity." (p. 236)

Grade 8 Mathematics, Quarter 1, Unit 1.3

Informally Proving Angle Properties

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Use manipulatives to establish the fact about the angles sum of triangles.
- Using informal arguments, establish facts about exterior angles of triangles.
- Create corresponding angles when parallel lines are cut by a transversal and make informal arguments that they are congruent.
- Construct alternate interior angles when parallel lines are cut by a transversal and make informal arguments that they are congruent.
- Generate alternate exterior angles when parallel lines are cut by a transversal and make informal arguments that they are congruent.

Mathematical practices to be integrated

Construct viable argument and critique the reasoning of others.

- Listen or read the facts others have developed over the angle sums and exterior angles of triangles. Decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Model with mathematics.

- Use manipulatives to create, construct, and generate angles when parallel lines are cut by a transversal.

Essential questions

- How do you show the informal argument of the measurement of angles between parallel lines cut by a transversal?
- What is the relationship of the exterior angle to the interior angles of a triangle?
- How many pairs of angles in a triangle have to be congruent to determine that the triangles are similar?
- How do I find the sum of the interior angles of a triangle?
- Without measuring angles, how can you tell if two triangles are similar?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**8.G**

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, ~~and the angle-angle criterion for similarity of triangles~~. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

In kindergarten, students began to identify and describe shapes. In grade 1, students reasoned with shapes and their attributes. In grade 2, students began to reason with angles and faces of different shapes. In grade 4, students drew right, acute, and obtuse angles as well as parallel lines.

In grade 5, students identified relationships between corresponding terms. Students also interpreted coordinate values of points in the context of the situation. In grade 6, students used rate language in the context of a ratio relationship. They understood the concept of a unit rate a/b associated with a ratio $a:b$. Sixth-grade students also solved unit rate problems involving unit pricing and constant speed.

In grade 7, students solved real-world and mathematical problems using numerical and algebraic expressions and equations. Students also solved word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$ and solved these equations fluently. Seventh-grade students also focused on constructing triangles from three measures of angles or sides, and they solved problems involving supplementary, complementary, vertical, and adjacent angles.

Current Learning

Understanding congruency and similarity using physical models, transparencies or geometry software is a major cluster in grade 8. Students use informal arguments to establish facts about the angle sum and mastery of exterior angles of triangles. Students continue to use informal arguments to establish facts about angles created by a transversal line cutting through parallel lines.

Students must do work with congruency and similarity before they are able to justify the connections among proportional relationships, lines, and linear equations which is a major cluster; therefore, this is a major focus for assessment.

Future Learning

In geometry, students will begin to formally prove theorems about lines and angles, triangles, and parallelograms. The students will also begin to make formal geometric constructions with a variety of tools and methods. Students will use these mathematical concepts on MapQuest, construction, and architecture.

Additional Findings

Students may have difficulty understanding the representations of linear functions. Therefore, they "should have frequent experiences in modeling situations with equations with the form $y = kx$, such as relating the side lengths and the perimeters of similar shapes." (*Principles and Standards for School Mathematics*, p. 227)

Grade 8 Mathematics, Quarter 1, Unit 1.4

Understanding and Using Rational Numbers, Irrational Numbers, and Radicals

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Identify that numbers that are not rational are called irrational (e.g., know that $\sqrt{2}$ is irrational).
- Understand informally that every number has a decimal expansion.
- Recognize rational numbers.
- Show that decimal expansions repeat eventually.
- Convert a decimal expansion that repeats eventually into a rational number.
- Use rational approximations of irrational numbers to compare between two rational whole numbers.
- Locate irrational numbers on a number line and estimate the value of expressions.
- Use square root and cube root symbols to represent solutions to equations of the form of $x^2 = p$ and $x^3=p$, where p is a positive rational number.
- Evaluate square roots of small perfect squares and cube roots of small perfect cubes.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Convert rational numbers, or approximate irrational numbers, as decimal expansions.

Model with mathematics.

- Model irrational numbers using a number line with whole number rational units.

Essential questions

- How do you order and compare rational and irrational numbers?
- What is the relationship between the side length and the area of a square?
- How would you use algebraic skills to simplify positive radical expressions in a problem situation?
- Why is it important to be able to approximate between which two whole numbers on a number line a given irrational number would be found?

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System	8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations	8.EE

Work with radicals and integer exponents.

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In kindergarten, students compared numbers between 1 and 10 as numerals. In grades 1–4, students developed an understanding of operations using whole numbers and fractions. In fifth grade, students integrated understanding of rational numbers as decimals.

In sixth grade, students multiplied and divided rational numbers in decimal form and located rational numbers on a number line. In seventh grade, students extended understanding of addition, subtraction, multiplication, and division of rational numbers.

Current Learning

“Know that there are numbers that are not rational, and approximate them by rational numbers” is a supporting cluster in eighth grade. In this unit, students understand informally that every number has a decimal representation. Students must find the fractional equivalent of a repeating decimal.

Students use a number line to compare rational and irrational numbers. They also find side lengths of squares and cubes by finding the square or cube root of the area. Students learn that a non-repeating, non-terminating decimal is an irrational number.

Future Learning

In algebra I, students will explain why the sum or product of two rational numbers is rational number. In geometry, students will use rational and irrational numbers to calculate length, area, and circumference of geometric figures.

In Statistics and Probability, students will use approximations to make inferences and draw conclusions. Calculations using rational and irrational numbers are used to prepare students for college level mathematics, calculate volume of natural gas in the petroleum industry, and for forecasting risk in the insurance industry.

Additional Findings

Students often struggle with converting decimals and fractions. Studies show that "the deepest translation problem in Pre-K to grade 8 concerns the translation between fractional and decimal representations of rational numbers." (*Adding It Up*, p. 101)

Grade 8 Mathematics, Quarter 1, Unit 1.5

Explaining and Applying the Pythagorean Theorem and Its Converse

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Develop a proof of the Pythagorean Theorem.
- Describe a proof of the converse of the Pythagorean Theorem.
- Utilize the Pythagorean Theorem to determine the unknown leg of a right triangle in a real-world mathematical problem in two-dimensions.
- Find the unknown hypotenuse of a right triangle in a real-world mathematical problem in two-dimensions using the Pythagorean Theorem.
- Use the Pythagorean Theorem to determine the unknown leg of a right triangle in a real-world mathematical problem in three dimensions.
- Discover the unknown hypotenuse of a right triangle in a real-world mathematical problem in three dimensions using the Pythagorean Theorem.
- Determine the distance between two points in a coordinate system using the Pythagorean Theorem.

Mathematical practices to be integrated

Model with mathematics.

- Look at a map, and find the distance between two places on it.

Use appropriate tools strategically.

- Use graphing technology, graph paper, and a straight edge to construct right triangles.

Look for and make use of structure.

- Recognize that to find the distance between two points, you have to square each leg, subtract them, and take the square root of that answer.

Essential questions

- How was the Pythagorean Theorem discovered?
- Where can you use the Pythagorean Theorem in the world?
- If you are given only a triangle's side lengths, how can you determine whether the triangle is a right triangle?
- How is distance related to the Pythagorean Theorem?
- What is the relationship between the legs and hypotenuse of a right triangle?
- Why is the converse of the Pythagorean Theorem important?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**8.G**

Understand and apply the Pythagorean Theorem.

8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grades 1–3, students reasoned with shapes and their attributes. In third grade, students represented and solved problems involving multiplication. In grade 4, students identified and classified shapes by properties of angles and used the four operations with whole numbers to solve problems.

In fifth grade, students continued to classify two-dimensional figures based on their properties and graphed points on the coordinate plane. They also performed multi-digit whole number multiplication and wrote and interpreted numerical expressions. In grade 6, students graphed and used coordinates to find distance between two points with one the two coordinates the same and mastered solving one-variable equations. In grade 7, students mastered multiplying rational numbers. Students also used multiplying rational numbers to generate equivalent expressions. Students, also, used algebraic expressions to solve real-life and mathematical problems.

Current Learning

Learning the Pythagorean Theorem is a major cluster in grade 8. Students explain a proof for Pythagorean Theorem and its converse. Students apply the Pythagorean Theorem to determine a leg length or the hypotenuse given the other two sides of a right triangle in real-world problems in two and three dimensions. Students also use multiplication to square numbers and to find the square roots of numbers.

Future Learning

In geometry, Pythagorean Theorem is going to be used to develop the distance formula. Also, students will use Pythagorean Theorem to define trigonometric ratios and solve problems involving right triangles. Pythagorean Theorem is used with polynomial identities to generate Pythagorean triples in algebra 2. Also, the Pythagorean Theorem is used to generate some trigonometric identities in algebra 2 and with the unit circle.

In careers, the converse of the Pythagorean Theorem is used in carpentry in roof framing, squaring foundations, and walls. Also, the Pythagorean Theorem is used in engineering and surveying to find lengths of different distances. Firemen, construction workers, and window washers often rely on the use of ladders in their line of work. They use the Pythagorean Theorem to discover if their ladder is long enough to reach what they need to.

Additional Findings

Using logical thinking to prove theorems is challenging for students since it requires them to think in a new and precise way about situations presented to them. Many students have difficulty with formulating justifications for their conclusions, especially in mathematics. "Proof is a form of justification, but not all justifications are proofs." (*Adding It Up*, p. 132)

Proofs can be challenging to students because of the need to justify reasoning to make meaning clear. Practice using a variety of techniques and perseverance help students to develop fluency in such reasoning. (*Adding It Up*, p. 280)

This content may be challenging for students since it requires them to justify reasoning in proof. *A Research Companion to Principles and Standards for School Mathematics* cites Thompson's (1966) definition of mathematical reasoning as "purposeful inference, deduction, induction and association in the areas of quantity and structure." (p. 228)

Adding It Up describes adaptive reasoning as "the capacity to think logically about the relationships among concepts and situations. Such reasoning is correct and valid, stems from careful consideration of alternatives, and includes knowledge of how to justify the conclusions." (p. 129)

This content is challenging for students due to the required use of tools and students' lack of experience using them. The use of manipulatives, tools, and technology supports higher level thinking that students will need in future development of proofs.

Wise use of manipulatives facilitates development of "sound representations of geometric concepts." (*A Research Companion to Principles and Standards for School Mathematics*, p. 155)

Technology helps to connect the construction skills and procedures to general mathematical development and concepts. (*Principles and Standards for School Mathematics*, p. 26)